Math Cheat Sheet Complex Numbers

Imaginary Unit.

It is the number $\sqrt{-1}$ and it is designated by i

Powers of i:

$i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i$ ...

Complex numbers:

These are those that can be written in the form $a + bi$, where a and b are real numbers and i the imaginary unit.

$a = \mathbf{Re}(z)$ is the real part

$b = \mathbf{Im}(z)$ is the imaginary part

The set of all complex numbers is denoted by $\mathbb{C}$ and is defined as $\mathbb{C} := \{z = a + bi / a, b \in \mathbb{R}\}$

Two complex numbers are equal when they have the same real part and the same imaginary part:

$a + bi = a' + b'i \Rightarrow a = a'$ and $b = b'$

Binomial form of a complex number:

$z = a + bi$

If $b = 0 \Rightarrow z = a$ is a real number.

If $b \neq 0$ and $a = 0 \Rightarrow z = bi$, it is a pure imaginary number.

Conjugate, opposite, and inverse of a number in binomial form:

- opposite: $-z = -a - bi$
- conjugate: $\bar{z} = a - bi$
- inverse: $z^{-1} = \frac{1}{z} = \frac{1}{a+bi}$

Representation of complex numbers

$\mathbb{C} := \{P(a, b) / a, b \in \mathbb{R}\}$

Each complex number $z = a + bi$ corresponds to the point $P(a, b)$, which is called the affix of the complex number.

If $z_1 = a + bi$ and $z_2 = c + di$, then:

Sum and subtraction of complex numbers:

- $z_1 + z_2 = (a + c) + (b + d)i$
- $z_1 - z_2 = (a - c) + (b - d)i$

Product of complex numbers:

The product of $z_1 \cdot z_2$ is obtained by multiplying each term of z_1 by each term of z_2 and applying the properties of the powers of i:

- $z_1 \cdot z_2 = (ac - bd) + (ad + bc)i$

Division of complex numbers:

To divide two complex numbers $\frac{z_1}{z_2}$, multiply the numerator and the denominator by the conjugate of the denominator:

- $\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z_2}}{z_2 \cdot \bar{z_2}}$

Complex Numbers in Polar Form: r_θ

$r = |z|$ is the modulus of a complex number.

θ is the argument of the complex number, which can be expressed in degrees or radians. Two complex numbers are equal if they have the same modulus and the same argument.

$r_\theta = r'_{\theta'} \Rightarrow r = r'$ and $\theta = \theta' + k \cdot 360°$, where $k \in \mathbb{Z}$

Always take $\theta \in [0, 360°)$

Conjugate, Opposite, and Inverse of a Number in Polar Form:

- opposite: $-z = r_{\theta+180°}$
- conjugate: $\bar{z} = r_{-\theta}$
- inverse: $z^{-1} = \frac{1}{z} = \frac{1}{r_\theta} = \left(\frac{1}{r}\right)_{-\theta}$

Conversion from Binomial to Polar Form:

$r = |z| = \sqrt{a^2 + b^2}$

$\tan\theta = \frac{b}{a} \Rightarrow \theta = \arctan\left(\frac{b}{a}\right)$

Conversion from Polar to Binomial Form:

$a = r\cos\theta$

$b = r\sin\theta$

Trigonometric Form:

$z = a + bi = r\cos\theta + (r\sin\theta)\,i = r\,(\cos\theta + i\sin\theta)$

Product of Complex Numbers in Polar Form:

$r_\theta \cdot r'_{\theta'} = (r \cdot r')_{\theta+\theta'}$

Quotient of Complex Numbers in Polar Form:

$\frac{r_\theta}{r'_{\theta'}} = \frac{r}{r'}{}_{\theta-\theta'}$

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